

Exercise 69

Is there a number that is exactly 1 more than its cube?

Solution

A number that is exactly 1 more than its cube satisfies

$$x = x^3 + 1.$$

Bring all terms to one side.

$$0 = x^3 - x + 1$$

The function $f(x) = x^3 - x + 1$ is a polynomial, so it's continuous everywhere on its domain by Theorem 7. Find a number x for which the function is negative, and find a number x for which the function is positive.

$$f(-2) = -5$$

$$f(-1) = 1$$

$f(x)$ is continuous on the closed interval $[-2, -1]$, and $N = 0$ lies between $f(-2)$ and $f(-1)$. By the Intermediate Value Theorem, then, there exists a root within $-2 < x < -1$. Therefore, there is a number that is exactly 1 more than its cube.